Research Statement - Elia Mazzucchelli

Over the past few years, I have learned and applied concepts at the intersection of theoretical physics and mathematics. I find this interplay both exciting and deeply fruitful: rich and often unexpected mathematical structures emerge naturally within fundamental physics, offering new insight into the underlying principles of physical theories. Motivated by this perspective, I am eager to pursue research directions that build on this connection and allow me to continue learning new ideas across both disciplines, with an eye toward understanding fundamental questions in theoretical physics.

Positive Geometries

Over the past two years, my main research focus has been on positive geometries. This interdisciplinary field emerged from the discovery of geometric structures in high-energy physics. Following the discovery of the amplituhedron [1], physicists identified various positive geometries that describe the scattering of particles in certain quantum field theories. Positive geometries are also mathematically intriguing, as they combine ideas from complex algebraic geometry and topology with real semialgebraic geometry, positivity, optimization, and combinatorics [2]. My research has been closely connected to a specific geometry in the Grassmannian, the amplituhedron, which encodes scattering amplitudes in $\mathcal{N} = 4$ super Yang–Mills (sYM) theory.

Positivity and Volumes

In [3], Hodges described certain gluon amplitudes in Yang–Mills theory as volumes of specific polytopes. The amplituhedron generalizes this construction to all tree-level amplitudes and loop-level integrands in $\mathcal{N} = 4$ super Yang–Mills (sYM) theory. The amplituhedron is canonically associated with a rational function (or differential form) that yields the amplitude (or its integrand); however, interpreting this function as a volume remains an open problem, which is often referred to as the *dual amplituhedron*.

Representing amplitudes as volumes is especially desirable because it makes manifest a strong positivity property called complete monotonicity. Recently, it has been observed that many quantities related to scattering amplitudes in various quantum field theories exhibit complete monotonicity [4]. In an ongoing project [5], we are investigating this property and its implications specifically for observables in $\mathcal{N} = 4$ sYM, in relation to the amplituhedron, see Figure 13.

In [6, 7], we approach the dual amplituhedron problem by identifying a class of positive geometries that admit a dual volume representation. This connects the notion of complete monotonicity to positive geometry. Our investigation provides a new dual description of positive geometries, in which the canonical rational differential form is replaced by a probability distribution on a dual geometry.

In [6], we propose a candidate semialgebraic set for the dual amplituhedron. This leads naturally to the study of a polytope, the *exterior cyclic polytope*, which generalizes the well-known cyclic polytope. We analyze the combinatorial structure of the exterior cyclic polytope, its facets, and their duals. To better understand the geometry of amplituhedra, we introduce a notion of convexity, called extendable convexity, for real semialgebraic sets in any embedded projective variety and study its implications for amplituhedra.

<u>*Future directions:*</u> Finding the dual amplituhedron is an ongoing project, which we expect will lead to a formulation of amplitudes in $\mathcal{N} = 4$ sYM as volumes of geometric objects in kinematic space. This formulation would allow us to explore its implications, including connections to integrability and the AdS/CFT correspondence. For instance, we anticipate that the dual amplituhedron may provide new insight into the string-like formulation of $\mathcal{N} = 4$ sYM [8]. More generally, the

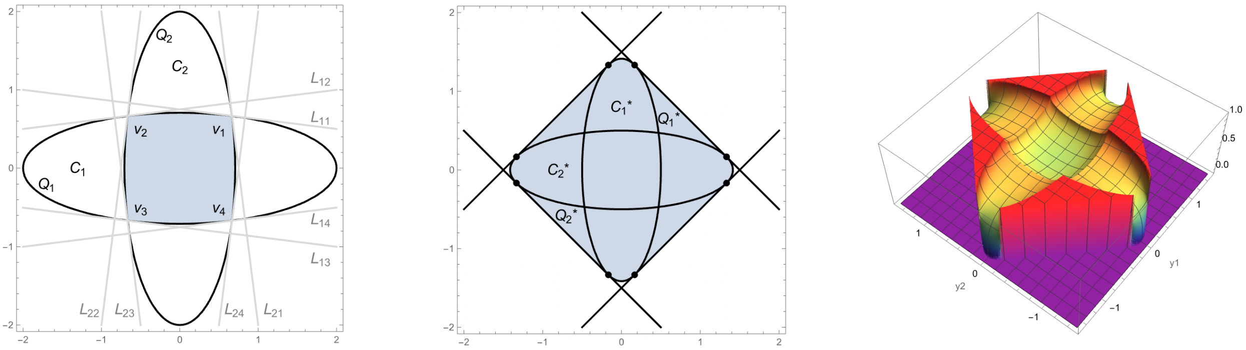

Figure 1: (Volumes) A positive geometry in the projective plane (left), its convex dual (middle) and the plot of the graph of the probability distribution supported on the dual (right), whose volume is the canonical function of the original geometry [6, Fig. 13].

dual volume description of positive geometries could offer a pathway for extending these ideas to other quantum field theories, where similar properties have been observed [4].

Singularity Structure

Understanding the analytic structure of observables in quantum field theories — specifically the type and location of singularities — provides insight into the physical properties of the theory. Ideally, this structure could be fully understood, allowing observables to be bootstrapped purely from their analytic properties. This approach is particularly promising in $\mathcal{N} = 4$ sYM, where the interplay between singularities and the positive geometry of the amplituhedron provides a powerful framework [9].

In this context, we study correlators of null, n-sided polygonal Wilson loops with a Lagrangian insertion in planar $\mathcal{N} = 4$ sYM, which admit a positive geometry description [10]. This finite observable is closely related to loop integrands of maximally-helicity-violating amplitudes in the same theory, and conjecturally to all-plus helicity amplitudes in pure Yang–Mills theory.

In [11], we fully classify the leading singularities of Wilson loops with Lagrangian insertions and prove that they have a hidden conformal symmetry. Leading singularities are rational functions that appear as prefactors in the decomposition of these observables into functions of uniform transcendental weight, and they correspond to maximal residues of the integrand. The boundary structure of the positive geometry constrains which iterative residues are accessible, allowing for an all-loop, all-point classification of these residues.

In [12], we leverage the interplay between positive geometry and Landau analysis, see Fig. 2. By associating Landau diagrams to geometric boundaries, we identify which solutions of the Landau equations are relevant as singularities of the integral. This approach provides an efficient method to determine the symbol alphabet for transcendental functions whose integrands admit a positive geometric description. Using this procedure, we successfully compute six-point two-loop and five-point three-loop contributions, which constitute building blocks for the Wilson loop with Lagrangian insertion.

The Landau analysis in momentum twistor variables exhibits a rich mathematical structure. In [13], we study varieties of lines in three-dimensional space arising in this context. We then apply these tools from algebraic geometry and combinatorics to advance the determination of leading singularities [14]. We also test a conjecture regarding the positivity of singularities in planar $\mathcal{N} = 4$ sYM, evaluating them on the configuration spaces of planar-ordered particles.

Future directions: Ideally, the positive geometry of the amplituhedron dictates the singularity structure of amplitudes in planar $\mathcal{N} = 4$ sYM. It is important to make this expectation more precise, for example by understanding the relationship between the positive geometry describing an integrand and the differential equations satisfied by the corresponding integral.

The amplituhedron encodes amplitudes in $\mathcal{N} = 4$ sYM as *canonical pairings* of a differential

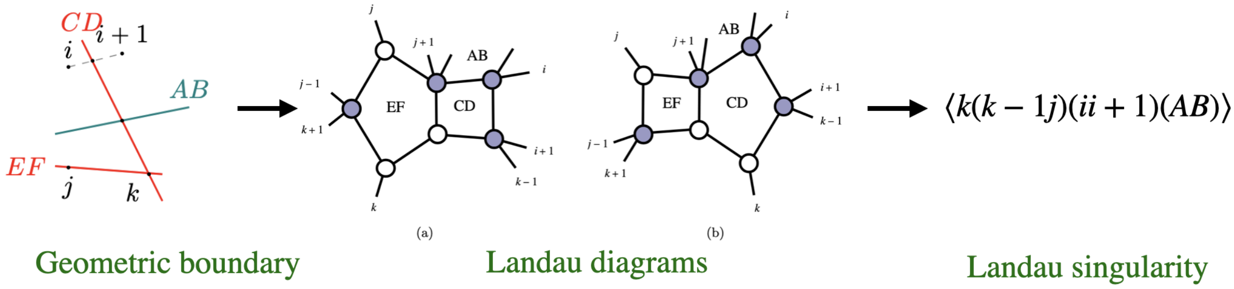

Figure 2: The positive geometry of the amplituhedron encodes the singularity structure of integrals appearing in $\mathcal{N} = 4$ sYM by the correspondence between geometric boundaries and Landau diagrams, refining the Landau analysis [12].

form with a semialgebraic cycle. To fully exploit the information contained in the amplituhedron, it is crucial to study the integration cycle. Doing so could provide a deeper geometric understanding of both the singularities and the differential equations satisfied by amplitudes. Another advantage of formulating amplitudes as canonical pairings is that they can, in principle, be efficiently evaluated numerically, making it very interesting to test this expectation in practice.

Geometries in the Coulomb Branch

In [15], we extend ideas from [16] and identify a larger class of positive geometries, constructed from deformations of the amplituhedron, which describe scattering amplitudes in the Coulomb branch of planar $\mathcal{N} = 4$ sYM theory. We compute scalar massive four-point two-loop planar diagrams depending on four scales, using these results to provide a family of Coulomb branch amplitudes that interpolate between known configurations in the literature. In this way, we establish both a positive geometry framework and a perturbative realization for a family of cusp anomalous dimensions computed via integrability.

Future directions: A natural extension is to explore the research directions discussed previously in the Coulomb branch setting, in connection with deformations of the amplituhedron.

Computational Algebraic Geometry

Positive geometries provide a rich mathematical framework, and their interplay between complex and real algebraic geometry makes them well suited for tools from computational algebraic geometry.

In [17], we study the variety given by the complement of hyperplanes in the Grassmannian. We provide a combinatorial formula for computing the Euler characteristic of this variety, both symbolically and numerically. This quantity is relevant in physics in the context of string-like amplitudes [8], as it encodes the algebraic complexity of solving likelihood (or scattering) equations.

In [18], we study a specific loop amplituhedron at two loops and four points. Using computational algebraic geometry, we determine the algebraic boundary stratification and its intersection with the real semialgebraic set. From this analysis, we prove that the numerator of the canonical form of this amplituhedron, the adjoint, is uniquely determined by interpolating the residual arrangement.

Future directions: I value the connection between theoretical physics and the mathematics community, from which I have learned over the past few years. I plan to further develop this connection by translating questions arising in physics into problems in mathematics. I also recognize the power of computational tools from algebraic geometry, both symbolic and numerical, which may have broad applications across physics.

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